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Polynomial Approximation Based Control for Nonlinear Systems

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Abstract This paper is concerned with the stabilization problem for nonlinear systems. A new polynomial-approximation-based approach for modeling nonlinear systems is first proposed. The nonlinearity is approximated by polynomials and the approximation errors are treated as modeling uncertainties. The original nonlinear systems are converted into polynomial systems with modeling uncertainties. In order to highlight the approximation accuracy, the piecewise polynomial approximation functions are utilized. A novel polynomial state-feedback controller is designed to solve the stabilization problem. Furthermore, switched polynomial state-feedback controllers are designed to improve the performance. The stabilization conditions are presented in terms of sum of squares, which can be numerically solved via SOSTOOLS. Finally, simulation examples are provided to demonstrate the feasibility of the proposed method and show its advantage over the polynomial-fuzzy-model-based approach.

Keywords Sum of squares(SOS) · Polynomial systems · Polynomial approximation systems · Switched controller

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1 Introduction

Due to the advancement of scientific technology, the real dynamic systems become more and more complicated. The nonlinearity is one of the reasons to lead to the complexity. Recently, various methods have been proposed to model and control the complexity in nonlinear systems [5, 6, 8, 23, 32, 33, 38, 45, 51]. The authors in [8] solved the state estimation problem of stochastic systems with switching measurements via proposing a novel polynomial approach. Fu et. al. [6] designed a novel global finite-time controller of a class of switched nonlinear systems with the powers of positive odd rational numbers and obtained some well performances. The authors in [33] proposed a novel filtering and fault detection approach for nonlinear systems with polynomial approximation. The nonlinear switched systems are composed of several subsystems and a switching law is employed to analyze a class of nonlinear hybrid systems [4, 10, 15, 21, 53]. The sliding mode control method has been introduced to stabilize nonlinear systems because of its virtues including fast response and good transient response [3, 9, 31, 46]. Furthermore, approximation-based adaptive control has been investigated via the backstepping technique for nonlinear systems [2, 7, 25, 42, 44]. The adaptive neural control and adaptive fuzzy control are utilized to handle the unknown nonlinear systems [14, 16, 26, 27, 34–37].

The Takagi-Sugeno (T-S) fuzzy model-based approach has received great attention for modeling nonlinear systems [17, 18, 20, 24, 30, 43, 45, 47, 49, 52]. The main reason is that it has been proved to be able to approximate any smooth nonlinear function to any degree of accuracy [1, 40]. Recently, the T-S fuzzy model was generalized into polynomial fuzzy model [41]. The polynomial fuzzy model inherits the virtues of T-S fuzzy model and always can represent the nonlinear systems using fewer fuzzy rules than the T-S fuzzy model, which reduces the complexity in sense [12, 13, 39]. Although T-S and polynomial fuzzy model based approaches have high ability on modeling nonlinear systems, there are drawbacks which can not be ignored. For example, the nonlinear systems can not be described by that in many situations, and it always requires more fuzzy rules to highlight the approximation accuracy, which leads to complexity and difficulty of analysis. Therefore, it is significant to find a simpler method to model nonlinear systems, which motivates this paper.

In this paper, a new method for modeling nonlinear systems which is named as polynomial-approximation-based approach is first proposed. The nonlinearities are approximated by polynomials and the approximation errors are treated as modeling uncertainties. After these processes, the original nonlinear systems are converted into polynomial systems with modeling uncertainties. Then, the robust control methods can be applied in our model. In order to highlight the approximation accuracy, piecewise polynomial approximation functions are employed. Polynomial state-feedback controller is designed to solve the stabilization problem, furthermore, switching polynomial state-feedback controllers are designed to improve the performance. The stabilization conditions are presented in terms of sum of squares, which can be numerically solved via SOSTOOLS. Finally, simulation examples are provided to demonstrate the

feasibility of the proposed method and show its advantage over the polynomial fuzzy model based approach.

The remaining part of this paper is organized as follows. Section 2 describes the features of SOS and develop the procedure to obtain the polynomial approximation systems. Section 3 proposes the design approach of the state-feedback controllers. Simulation examples are exploited to demonstrate the feasibility of the proposed approaches in Section 4 and Section 5 concludes this paper.

Notation: Some notations are employed in the paper. For instance, “ $*$ ” represents the transposed elements of a symmetric matrix. The symbol I denotes the identity matrices with appropriate dimensions. The matrix transpose and inverse will be written as the superscripts “ T ” and “ -1 ” respectively.

2 Modeling of Polynomial Approximation Systems

In this section, the polynomial-approximation-based approach for modeling nonlinear systems is proposed. The SOS-based stability analysis conditions for polynomial nonlinear systems are derived.

2.1 Sum of squares

In this paper, the SOS decomposition of multivariate polynomials is employed as the computational method. A multivariate polynomial $f(x(t))$ satisfies $f(x(t)) = \sum_{j=1}^r g_j(x(t))^2$, e.g., $x_1(t)^2 + 2x_1(t) + 1 = (x_1(t) + 1)^2$ is referred as sum of squares. Obviously, $f(x(t)) \geq 0$ if $f(x(t))$ is a SOS.

Definition 1 For a polynomial $f(x(t))$ in $x(t) \in R^n$ of degree $2d$, and $\hat{x}(x(t))$ with degree no greater than d , where $\hat{x}(x(t)) \in R^n$ is a column vector of monomials in $x(t)$. Then, a SOS with multivariate structure can be defined as $f(x(t)) = \hat{x}^T(x(t)) P \hat{x}(x(t))$, where $P \geq 0$.

The algorithm of the SOS decomposition for $f(x(t))$ is the semidefinite programming. Its objective is to find such P in the cone of positive semidefinite matrices.

2.2 Polynomial approximation systems

Consider the following nonlinear system:

$$\dot{x}(t) = A(t)\hat{x}(x(t)) + B(t)u(t), \quad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $A(t) \in R^{n \times n}$ is the system matrix, and $B(t) \in R^{n \times m}$ is the input matrix. The term $\hat{x}(x(t)) \in R^n$ is a column vector of monomials in $x(t)$. The monomial in

$x(t)$ is defined as $\prod_{s=1}^n x_s^{d_s}(t)$, where $x_s(t)$ denotes the element of vector $x(t)$, and d_s are known positive integers which denote the degree of each element.

It is difficult to handle the stability and stabilization problems directly due to the highly complexity of nonlinear systems. It can be known that the nonlinearities in system (1) can be handled in the form of polynomials via the SOS-based approach. Next, the system (1) can be rewritten as follows:

$$\begin{aligned} \dot{x}(t) = & [A(x(t)) + A_p(x(t)) + \Delta A_p(x(t))] \hat{x}(x(t)) \\ & + [B(x(t)) + B_p(x(t)) + \Delta B_p(x(t))] u(t), \end{aligned} \quad (2)$$

where $A(x(t))$, $B(x(t))$ are the convex part of $A(t)$ and $B(t)$, respectively, which contain linear and polynomial parts; $A_p(x(t))$, $B_p(x(t))$ denote the polynomial approximation of the non-convex part of $A(t)$, $B(t)$, respectively, and $\Delta A_p(x(t))$, $\Delta B_p(x(t))$ are the approximation errors, respectively.

The polynomial-approximation-based approach performs well in approximating nonlinearities. However, the same polynomial approximation function is used in the overall state space, which would lead to estimation errors unavoidably. In order to reduce the approximation errors, the state space is divided into some sub-regions, in which different polynomial approximation functions are used in different sub-regions.

Consider the system state $x(t) = [x_1(t) \ x_2(t) \ \cdots \ x_n(t)]^T$, $x(t) \in \Omega$, where Ω is the known bounded n -dimensional state space. For every state $x_\theta(t)$, $\theta = 1, 2, \dots, n$, we divide $x_\theta(t)$ into ω_θ connected sub-regions. Thus, the overall state space Ω is divide into $l = \prod_{\theta=1}^n \omega_\theta$ sub-state spaces and we have $\Omega = \cup_{i=1}^l \Omega_i$, where Ω_i is one of the sub-state spaces.

Then, we can get the following representation:

$$A_p(x(t)) + \Delta A_p(x(t)) = \cup_{i=1}^l (A_{pi}(x(t)) + \Delta A_{pi}(x(t))), \quad (3)$$

where $A_{pi}(x(t))$, $\Delta A_{pi}(x(t))$ are the polynomial approximation function and the approximation error in every sub-state space Ω_i , respectively.

Remark 1 Theoretically, the number of sub-state spaces are in direct proportion to the approximation accuracy. For the appropriate approximation algorithm and number of sub-state spaces, the proposed approach will not lead to approximation errors, i.e., $\Delta A_{pi}(x(t)) = 0$. In other words, the polynomial approximation systems can approximate the original nonlinear systems to any degree of accuracy.

Remark 2 It can be known that the system in (2) can represent a class of nonlinear systems including the system in (1). If $\Delta A_{pi}(x(t)) = 0$, the system in (1) and the system in (2) represent the same system.

To facilitate the stability analysis, we adopt the following conversion for formula (3):

$$A_p(x(t)) + \Delta A_p(x(t)) = \sum_{i=1}^l \beta_i (A_{pi}(x(t)) + \Delta A_{pi}(x(t))), \quad (4)$$

where

$$\beta_i = \begin{cases} 1 & x(t) \in \Omega_i \\ 0 & \text{otherwise} \end{cases},$$

the introduced β_i is adopted as the switching law for the polynomial approximation system. When the system states fall into one of the sub-state spaces, the corresponding sub-polynomial approximation system will be activated. Similarly, applying the same conversion to $B_p(x(t)) + \Delta B_p(x(t))$, we have

$$B_p(x(t)) + \Delta B_p(x(t)) = \sum_{i=1}^l \beta_i (B_{pi}(x(t)) + \Delta B_{pi}(x(t))). \quad (5)$$

On the other hand, denote

$$\left[\sum_{i=1}^l \beta_i \Delta A_{pi}(x(t)) \quad \sum_{i=1}^l \beta_i \Delta B_{pi}(x(t)) \right] = N \sum_{i=1}^l \beta_i F_i(t) [Q_1 \quad Q_2],$$

where N , Q_1 , Q_2 are known matrices with appropriate dimensions. Define $Z(t) = \sum_{i=1}^l \beta_i F_i(t)$ ($i = 1, 2, \dots, l$), then we know that $Z(t)$ is a time-varying matrix with measurable elements, which depend on the system states and satisfy $Z^T(t)Z(t) \leq I$.

Remark 3 Since the details of the approximation errors are not useful for the stability analysis, we can treat the switching error functions as a black box, that is, only the property of $Z^T(t)Z(t) \leq I$ will be employed and the details of the switching laws for approximation errors are not considered.

Based on aforementioned discussions, we get the following polynomial approximation system for system in (1):

$$\begin{aligned} \dot{x}(t) = & \sum_{i=1}^l \beta_i \{ [A(x(t)) + A_{pi}(x(t)) + NZ(t)Q_1] \hat{x}(x(t)) \\ & + [B(x(t)) + B_{pi}(x(t)) + NZ(t)Q_2] u(t) \}. \end{aligned} \quad (6)$$

Remark 4 If the approximation error part is ignored, the system in (6) will become a piecewise system. One can see that, by the above modeling approach, the nonlinear system was converted into a polynomial system with uncertainties which can be solved via the SOS based approach and the robust control approach.

2.3 Algorithm for polynomial approximation

There are many classical methods for approximating nonlinear functions by polynomials, such as Chebyshev polynomial approximation and Legendre polynomial approximation, as well as algorithms to highlight the approximation accuracy, such as Remez algorithm. Furthermore, there are also novel methods

developed in recent years [11, 50]. In this paper, we will use the Taylor series approach to obtain the polynomial approximation system which is expressed in the following lemma.

Lemma 1 *If arbitrary function $f(\mathbf{x})$ has $\mu+1$ order continuous partial derivatives on the neighborhood $\cup P(\mathbf{x}_0)$ of point $P(\mathbf{x}_0)$ with respect to the independent variable \mathbf{x} , where $\mathbf{x} = (x_1, x_2, \dots, x_n)$. Then for each point in $\cup P(\mathbf{x}_0)$, there exists a arbitrary scalar α such that the following equality holds:*

$$f(\mathbf{x}) = \sum_{k=0}^{\mu} \frac{1}{k!} \left(\sum_{i=1}^n (x_i - x_{i0}) \frac{\partial}{\partial x_i} \right)^k f(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_0} + L(\mathbf{x}, \alpha),$$

where $L(\mathbf{x}, \alpha) = \frac{1}{(\mu+1)!} \left(\sum_{i=1}^n (\alpha x_i - x_{i0}) \frac{\partial}{\partial x_i} \right)^{\mu+1} f(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_0}$ is the Lagrange remainder term.

Remark 5 It is known that, for fixed number of sub-state spaces, as μ goes to large, the Lagrange remainder term $L(\mathbf{x}, \alpha)$ will go to small, particularly, $\mu \rightarrow \infty$, $L(\mathbf{x}, \alpha) \rightarrow 0$. Furthermore, the approximation error for system (1) goes to zero.

Remark 6 From Lemma 1, it is clear that the proposed method can deal with nonlinear systems with the system matrix being a function all of x_1, x_2, \dots, x_n . However, the algorithm for polynomial approximation may be more complicated and the computational complexity of simulation will increase as the variables in nonlinear functions increase.

To obtain the main results in this paper, the following lemma is introduced.

Lemma 2 [48] *Giving matrices $\Sigma_1, \Sigma_2, \Sigma_3$ with appropriate dimensions satisfying $\Sigma_1^T = \Sigma_1$, then the following two inequalities are equivalent:*

$$\begin{aligned} \Sigma_1 + \Sigma_3 F(t) \Sigma_2 + \Sigma_2^T F^T(t) \Sigma_3^T &< 0, \\ \Sigma_1 + \varepsilon^{-1} \Sigma_3 \Sigma_3^T + \varepsilon \Sigma_2^T \Sigma_2 &< 0, \end{aligned} \quad (7)$$

where $F(t) F^T(t) < I$, and the scalar $\varepsilon > 0$.

3 Design of State-feedback Controllers

In this section, the state-feedback controller will be designed for the polynomial approximation system (6).

3.1 Polynomial state-feedback controller

The following polynomial state-feedback controller is considered:

$$u(t) = K(x(t)) \hat{x}(x(t)), \quad (8)$$

where $K(x(t)) \in R^{n \times m}$ is the state-feedback gain matrix. Thus we have the closed-loop system as follows:

$$\begin{aligned} \dot{x}(t) = & \sum_{i=1}^l \beta_i \{ [A(x(t)) + A_{pi}(x(t)) + NZ(t)Q_1] \hat{x}(x(t)) \\ & + [B(x(t)) + B_{pi}(x(t)) + NZ(t)Q_2] K(x(t)) \hat{x}(x(t)) \}. \end{aligned} \quad (9)$$

Theorem 1 *For a certain division of the concerned state space, and an arbitrary approximation algorithm explained in Section II, if there exist the symmetric matrix $X > 0$, polynomial matrices $G(x)$ with appropriate dimensions, and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that the following conditions hold:*

$$v_1^T (X - \varepsilon_1) v_1 \text{ is SOS}, \quad (10)$$

$$-v_2^T \left(\begin{bmatrix} \Psi(x) + \Psi^T(x) & * & * \\ Q_1 X & -\varepsilon_1 I & * \\ Q_2 G(x) & 0 & -\varepsilon_2 I \end{bmatrix} + \varepsilon_2(x) \right) v_2 \text{ is SOS}, \quad (11)$$

where v_1, v_2 are arbitrary vectors independent of $x(t)$; ε_1 is a nonnegative matrix, $\varepsilon_2(x)$ is a nonnegative polynomial matrix, and

$$\begin{aligned} \Psi(x) &= H(x) \bar{A}_i(x) X + H(x) \bar{B}_i(x) G(x) + (\varepsilon_1 + \varepsilon_2) H(x) N N^T H^T(x), \\ \bar{A}_i(x) &= A(x) + A_{pi}(x), \quad \bar{B}_i(x) = B(x) + B_{pi}(x), \end{aligned}$$

in which $A_{pi}(x), B_{pi}(x)$ are polynomial matrices determined by the approximation algorithm, N, Q_1, Q_2 are arbitrary matrices to be determined, and $H(x)$ is a polynomial matrix with (i, j) th entry given by:

$$H_{ij}(x) = \frac{\partial \hat{x}_i}{\partial x_j}(x), \quad (12)$$

then the system represented by (9) is asymptotically stable. The gain matrices are obtained as $K(x) = G(x) X^{-1}$.

Proof Construct the polynomial Lyapunov function as follows:

$$V(t) = \hat{x}^T(x(t)) P \hat{x}(x(t)). \quad (13)$$

In order to simplify the notations, we will drop the variable t in the following discussion, for example, $\hat{x}(x) = \hat{x}(x(t))$. According to the trajectories of system (9), the time derivative of $V(t)$ can be obtained:

$$\begin{aligned} \dot{V}(t) &= \dot{\hat{x}}^T(x) P \hat{x}(x) + \hat{x}^T(x) P \dot{\hat{x}}(x) \\ &= \dot{x}^T H^T(x) P \hat{x}(x) + \hat{x}^T(x) P H(x(t)) \dot{x} \\ &= \sum_{i=1}^l \beta_i \hat{x}^T(x) [\Phi(x) + \Phi^T(x)] \hat{x}(x), \end{aligned}$$

where $H(x)$ is given in (12), and

$$\begin{aligned}\Phi(x) &= PH(x)\bar{A}_i(x) + PH(x)NZ(t)Q_1 \\ &\quad + PH(x)\bar{B}_i(x)K(x) + PH(x)NZ(t)Q_2K(x), \\ \bar{A}_i(x) &= A(x) + A_{pi}(x), \bar{B}_i(x) = B(x) + B_{pi}(x).\end{aligned}$$

Because of $\beta_i \geq 0$, thus $\dot{V}(t) \leq 0$ can be guaranteed by

$$\Phi(x) + \Phi^T(x) \leq 0, \quad (14)$$

for all $i = 1, 2, \dots, l$.

Pre- and post- multiplying (14) by P^{-1} and P^{-T} , recalling Lemma 2, employing the Schur complement, and denoting $X = P^{-1}$, $G(x) = K(x)X$, we know that $\dot{V}(t) \leq 0$ if the following inequality holds:

$$\begin{bmatrix} \Psi(x) + \Psi^T(x) & * & * \\ Q_1X & -\varepsilon_1 I & * \\ Q_2G(x) & 0 & -\varepsilon_2 I \end{bmatrix} \leq 0,$$

where $\Psi(x) = H(x)\bar{A}_i(x)X + H(x)\bar{B}_i(x)G(x) + (\varepsilon_1 + \varepsilon_2)H(x)NN^TH^T(x)$. The proof is completed.

Remark 7 According to Theorem 1, the number of decision variables is 4, the number of SOS conditions is $l + 1$, and the number is only determined by the number of sub-state spaces, but independent of the number of nonlinearities, which is different from the fuzzy-model-based approach for stabilization analysis.

3.2 Switching polynomial state-feedback controllers

Referring to Theorem 1, a common polynomial controller is designed for each polynomial approximation system. In order to reduce the conservativeness, switching polynomial state-feedback controllers will be developed for each polynomial approximation system. When the system states fall into one of the sub-state spaces, the polynomial controller corresponding to the sub-state space will be activated. Since all the states are measurable here, the system states which determine the sub-state spaces can be employed as the switching laws for the switching polynomial state-feedback controllers. Therefore, we develop the switching polynomial state-feedback controller as follows:

$$\begin{aligned}u(t) &= K(x(t))\hat{x}(x(t)) \\ &= \sum_{i=1}^l \beta_i K_i(x(t))\hat{x}(x(t)).\end{aligned}$$

Then we have the closed-loop system as follows:

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^l \beta_i \{ [A(x(t)) + A_{pi}(x(t)) + NZ(t)Q_1] \hat{x}(x(t)) \\ &\quad + [B(x(t)) + B_{pi}(x(t)) + NZ(t)Q_2] K_i(x(t)) \hat{x}(x(t)) \}. \quad (15)\end{aligned}$$

Theorem 2 For a certain division of the concerned state space, and an arbitrary approximation algorithm explained in Section II, if there exist symmetric matrix $X > 0$, polynomial matrices $G_i(x)$ ($i = 1, 2, \dots, l$) with appropriate dimensions, and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that the following conditions hold:

$$v_1^T (X - \varepsilon_1) v_1 \text{ is SOS,} \quad (16)$$

$$-v_2^T \left(\begin{bmatrix} \Psi(x) + \Psi^T(x) & * & * \\ Q_1 X & -\varepsilon_1 I & * \\ Q_2 G_i(x) & 0 & -\varepsilon_2 I \end{bmatrix} + \varepsilon_2(x) \right) v_2 \text{ is SOS,} \quad (17)$$

where v_1, v_2 are arbitrary vectors independent of $x(t)$; ε_1 is a nonnegative matrix, $\varepsilon_2(x)$ is a nonnegative polynomial matrix, and

$$\begin{aligned} \Psi(x) &= H(x) \bar{A}_i(x) X + H(x) \bar{B}_i(x) G_i(x) + (\varepsilon_1 + \varepsilon_2) H(x) N N^T H^T(x), \\ \bar{A}_i(x) &= A(x) + A_{pi}(x), \quad \bar{B}_i(x) = B(x) + B_{pi}(x), \end{aligned}$$

in which $A_{pi}(x), B_{pi}(x)$ are polynomial matrices determined by the approximation algorithm, N, Q_1, Q_2 are arbitrary matrices to be determined, and $H(x)$ is a polynomial matrix with (i, j) th entry given by:

$$H_{ij}(x) = \frac{\partial \hat{x}_i}{\partial x_j}(x), \quad (18)$$

then the system represented by (9) is asymptotically stable. And the controller gain matrices are obtained as $K_i(x) = G_i(x) X^{-1}$, thus, the switching polynomial state-feedback controller can be summarized as $K(x) = K_i(x), x \in \Omega_i$.

Proof The proof is omitted.

Remark 8 According to Theorem 2, the number of decision variables is $l + 3$, the number of SOS conditions is $l + 1$ which is the same as that in Theorem 1. And Theorem 2 can be reduced to Theorem 1 readily by using one controller.

4 Simulation Results

An example is used to demonstrate the effectiveness of the proposed state-feedback controller design approach in this section. Consider the following nonlinear system:

$$\dot{x}(t) = \begin{bmatrix} 4.5 + \frac{1}{1+e^{-(x_1+2)}} & 1.2x_1^2 + 0.1 \\ 2.5 & -3 \end{bmatrix} \hat{x}(x(t)) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t), \quad (19)$$

where $\hat{x}(x(t)) = [x_1(t) \ x_2(t)]^T$. Obviously, it is non-convex in $\frac{1}{1+e^{-(x_1+2)}}$, and it only depends on x_1 . According to Section II, the concerned known bounded state space is assumed as $\Omega = \{x_1 | x_1 \in [-2, 2]\}$.

Case 1: At first, we divide x_1 into 4 subregions with the interval of each subregion being 1, i.e., $[-2, -1], (-1, 0], (0, 1], (1, 2]$, thus, the overall state

space is divided into $l = 4$ sub-state spaces accordingly. Referring to Lemma 1, the truncation order $\mu + 1$ is chosen as 3, the expansion points are chosen as $[-1.5, -0.5, 0.5, 1.5]$. Based on the division, we obtain the polynomial approximation system as follows:

$$\dot{x}(t) = \sum_{i=1}^l \beta_i [A(x(t)) + H A_{pi}(x(t)) H^T + N Z(t) Q_1] \hat{x}(x(t)) + B(x(t)) u(t),$$

where

$$A(x(t)) = \begin{bmatrix} 4.5 & 1.2x_1^2 + 0.1 \\ 2.5 & -3 \end{bmatrix}, \quad B(x(t)) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad H = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

and $A_{pi}(x(t))$ is the polynomial approximation of $\frac{1}{1+e^{-(x_1+2)}}$ in sub-state space Ω_i , for example, when x_1, x_2 fall into $\Omega_1 = \{x_1 | x_1 \in [-2, -1]\}$, $A_{p1}(x(t))$ is the Taylor expansion of $\frac{1}{1+e^{-(x_1+2)}}$ with expansion points $x_1 = -1.5$. $A_{pi}(x(t))$ can be computed as

$$\begin{aligned} A_{p1}(x(t)) &= 0.2350x_1 - 0.0288(x_1 + 1.5)^2 + 0.9750, \\ A_{p2}(x(t)) &= 0.1491x_1 - 0.0474(x_1 + 0.5)^2 + 0.8921, \\ A_{p3}(x(t)) &= 0.0701x_1 - 0.0297(x_1 - 0.5)^2 + 0.8891, \\ A_{p4}(x(t)) &= 0.0285x_1 - 0.0134(x_1 - 1.5)^2 + 0.9280. \end{aligned}$$

In addition, choosing $N = [0.25 \ 0]^T$, $Q_1 = [0.1 \ 0]$, and ε_1 is treated as an unknown variable. According to Theorem 1, using the Matlab Toolbox SOSTOOLS, the state-feedback controller gain matrix can be calculated as follows:

$$K(x(t)) = [-5.9982x_1^2 - 0.0594x_1 - 11.2794, 0.0342x_1^2 + 0.0031x_1 + 1.7083],$$

Set the initial conditions of states as $x(t) = [-1 \ -2]^T$. Fig. 1 plots the state response of the closed-loop system under the polynomial state-feedback controller, and the control signal of the controller is shown in Fig. 2. One can see that, under the effect of the designed polynomial state-feedback controller, the system goes to stable at about 2.4s.

Remark 9 Concerning the determination of matrices N and Q_1 , we should guarantee the time-varying matrix $Z^T(t)Z(t) \leq I$. In this example, the maximum error between the nonlinearity $\frac{1}{1+e^{-(x_1+2)}}$ and its approximation is 0.0022, so that the chosen of N and Q_1 , should satisfy the following conditions

$$\begin{aligned} \Delta A_p(x(t))^{11} &= N^{11} Z(t) Q_1^{11} \leq 0.0022, \\ Z^T(t) Z(t) &\leq I, \end{aligned}$$

where N^{11} , Q_1^{11} , $\Delta A_p(x(t))^{11}$ are the elements at the first row and first column of N , $\Delta A_p(x(t))$ and Q_1 , respectively.

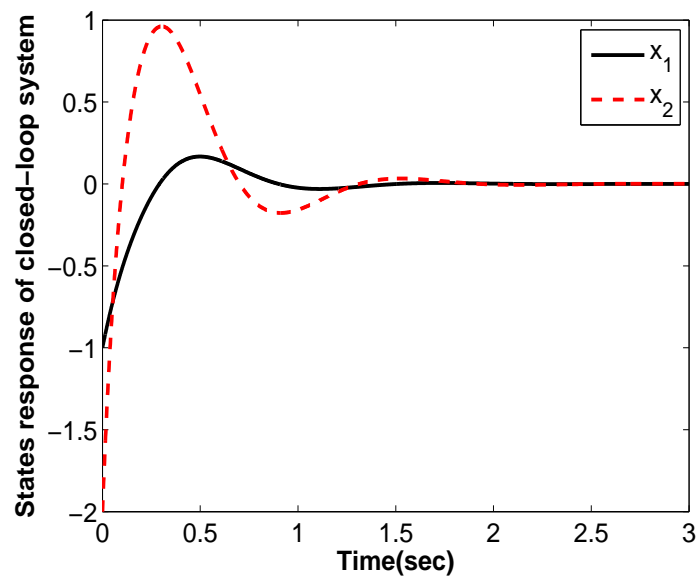


Fig. 1 The state response.

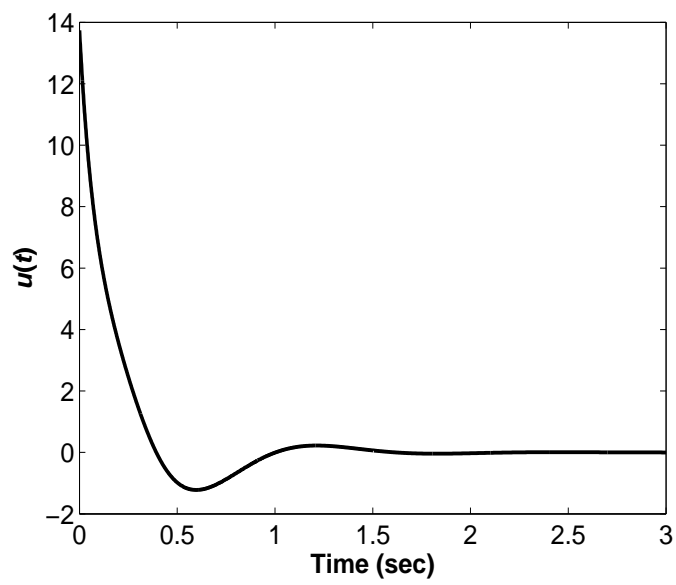


Fig. 2 The control signal.

Case 2: We equally divide $x_1 \in [-2, 2]$ into 8 sub-regions with the interval of each sub-region being 0.5. Referring to Lemma 1, the truncation order $\mu + 1$ is also chosen as 3, the expansion points are chosen as $[-1.75, -1.25, -0.75, -0.25, 0.25, 0.75, 1.25, 1.75]$. Thus, the overall state space is divided into $l = 8$ sub-state spaces. Based on this division, we have the polynomial approximation system as follows:

$$\dot{x}(t) = \sum_{i=1}^l \beta_i [A(x(t)) + A_{pi}(x(t)) + NZ(t)Q_1] \hat{x}(x(t)) + B(x(t))u(t),$$

where

$$A(x(t)) = \begin{bmatrix} 4.5 & 1.2x_1^2 + 0.1 \\ 2.5 & -3 \end{bmatrix}, \quad B(x(t)) = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

and $A_{pi}(x(t))$ is the polynomial approximation of $\frac{1}{1+e^{-(x_1+2)}}$ in sub-state space Ω_i , for example, when x_1, x_2 fall into $\Omega_1 = \{x_1 | x_1 \in [-2, -1.5]\}$, $A_{p1}(x(t))$ is the Taylor expansion of $\frac{1}{1+e^{-(x_1+2)}}$ with expansion points $x_1 = -1.75$. $A_{pi}(x(t))$ can be computed as

$$\begin{aligned} A_{p1}(x(t)) &= 0.2461x_1 - 0.0153(x_1 + 1.75)^2 + 0.9929, \\ A_{p2}(x(t)) &= 0.2179x_1 - 0.0390(x_1 + 1.25)^2 + 0.9515, \\ A_{p3}(x(t)) &= 0.1731x_1 - 0.0480(x_1 + 0.75)^2 + 0.9071, \\ A_{p4}(x(t)) &= 0.1261x_1 - 0.0444(x_1 + 0.25)^2 + 0.8835, \\ A_{p5}(x(t)) &= 0.0863x_1 - 0.0349(x_1 - 0.25)^2 + 0.8831, \\ A_{p6}(x(t)) &= 0.0565x_1 - 0.0248(x_1 - 0.75)^2 + 0.8976, \\ A_{p7}(x(t)) &= 0.0359x_1 - 0.0166(x_1 - 1.25)^2 + 0.9178, \\ A_{p8}(x(t)) &= 0.0224x_1 - 0.0107(x_1 - 1.75)^2 + 0.9377. \end{aligned}$$

In addition, choosing $N = [0.25 \ 0]^T$, $Q_1 = [0.1 \ 0]$, according to Theorem 1, using the Matlab Toolbox SOSTOOLS, the state-feedback controller gain matrix can be calculated as follows:

$$K(x(t)) = [-6.6718x_1^2 - 0.0638x_1 - 12.3301, 0.0939x_1^2 + 0.0034x_1 + 1.8021],$$

Set the initial conditions of states as $x(t) = [-1 \ -2]^T$. Fig. 3 plots the state response of the closed-loop system with the polynomial state-feedback controller, and the control signal of the controller is shown in Fig. 4. Obviously, the closed-loop system goes to stable at about 1.7s.

Case 3: In this case, we will discuss the effectiveness of the switching polynomial state-feedback controller. Apply the same division and choose the same parameters as Case 2. According to Theorem 2, using the Matlab Toolbox SOSTOOLS, the switching state-feedback controller gain matrices can be calculated as follows:

$$\begin{aligned} K_1(x(t)) &= [-10.1731x_1^2 - 0.0829x_1 - 15.4984, \\ &\quad -0.4067x_1^2 + 0.8559 \times 10^{-4}x_1 + 1.0123], \end{aligned}$$

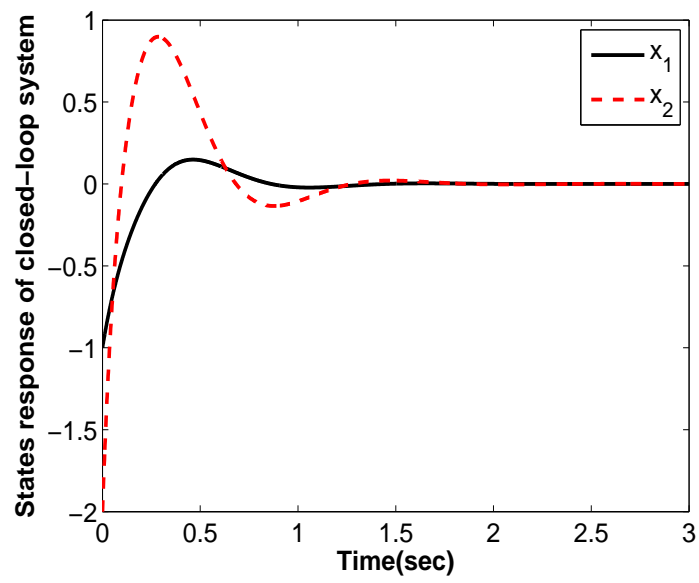


Fig. 3 The state response.

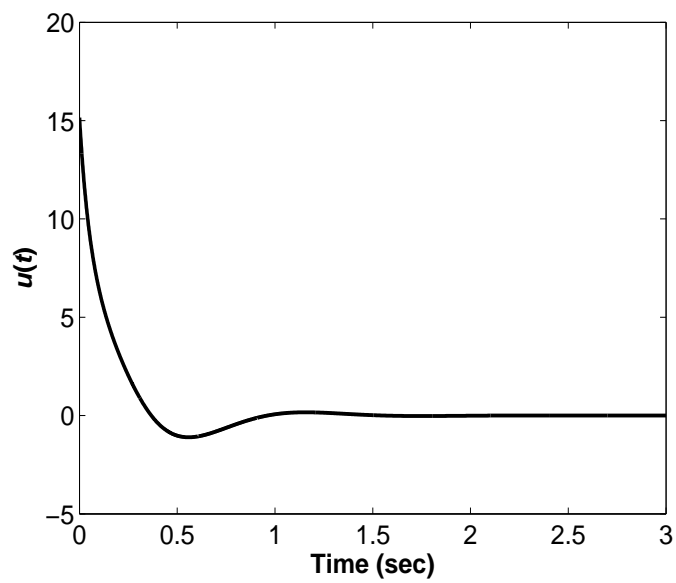


Fig. 4 The control signal.

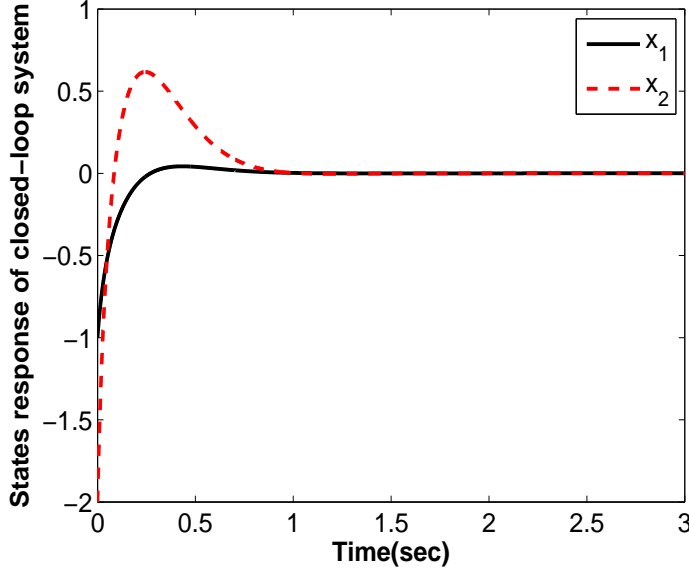


Fig. 5 The state response.

$$\begin{aligned}
 K_2(x(t)) &= [-10.1649x_1^2 - 0.0517x_1 - 15.4748, \\
 &\quad -0.4067x_1^2 + 0.5428 \times 10^{-4}x_1 + 1.0122], \\
 K_3(x(t)) &= [-10.1618x_1^2 - 0.0434x_1 - 15.4704, \\
 &\quad -0.4067x_1^2 + 0.4579 \times 10^{-4}x_1 + 1.0122], \\
 K_4(x(t)) &= [-10.1630x_1^2 - 0.0447x_1 - 15.4706, \\
 &\quad -0.4067x_1^2 + 0.4704 \times 10^{-4}x_1 + 1.0122], \\
 K_5(x(t)) &= [-10.1663x_1^2 - 0.0446x_1 - 15.4707, \\
 &\quad -0.4067x_1^2 + 0.4684 \times 10^{-4}x_1 + 1.0122], \\
 K_6(x(t)) &= [-10.1697x_1^2 - 0.0403x_1 - 15.4719, \\
 &\quad -0.4067x_1^2 + 0.4224 \times 10^{-4}x_1 + 1.0122], \\
 K_7(x(t)) &= [-10.1725x_1^2 - 0.0333x_1 - 15.4753, \\
 &\quad -0.4067x_1^2 + 0.3481 \times 10^{-4}x_1 + 1.0122], \\
 K_8(x(t)) &= [-10.1745x_1^2 - 0.0258x_1 - 15.4809, \\
 &\quad -0.4067x_1^2 + 0.2682 \times 10^{-4}x_1 + 1.0122],
 \end{aligned}$$

when $x(t)$ falls into one of the sub-state spaces, the corresponding controllers are activated. Fig. 5 plots the state response of the closed-loop system with the polynomial state-feedback controller, and the control signal of the controller is shown in Fig. 6. It is clear that the closed-loop system goes to stable at about 1s.

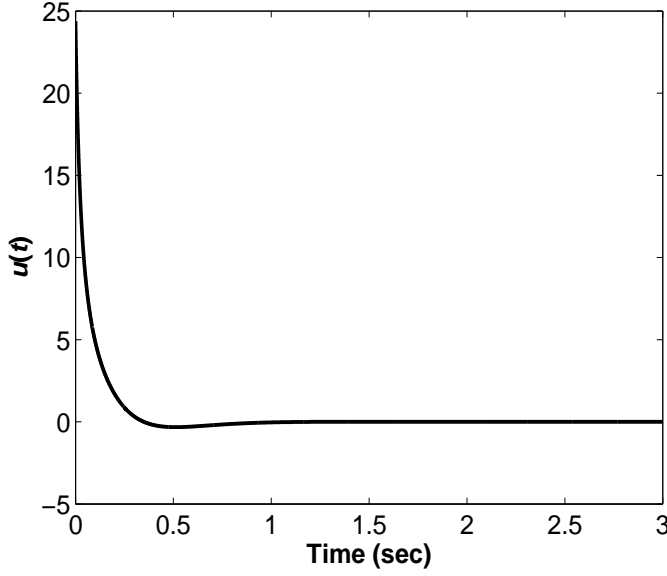


Fig. 6 The control signal.

Remark 10 One can see that, in this example, the nonlinearity in the nonlinear system is only dependent on x_1 , so that the division of x_2 has no contribution to highlight the approximation accuracy. In other words, the division of x_2 is unnecessary to the stabilization analysis and x_2 does not need to have bounds.

Remark 11 In the existing results, the switching control approach has been proved to have a better applicability than the traditional one. In this paper, the advantage of switching polynomial state-feedback controller has been shown in two-order systems in simulation. For the high-dimensional nonlinear systems, the switching polynomial state-feedback controller still has a better ability prompting system stability than the polynomial state-feedback controller.

Case 4: In this case, we will illustrate the advantages of the proposed method over the fuzzy-model-based approach. By applying the sector nonlinearity method, considering $x_1 \in [-2, 2]$, the dynamic of system (19) can be converted into the following polynomial fuzzy model:

Plant Rule 1 : IF $x_1(t)$ is about -2 , THEN

$$\dot{x}(t) = A_1(x(t))\hat{x}(x(t)) + B_1(x(t))u(t),$$

Plant Rule 2 : IF $x_1(t)$ is about 2 , THEN

$$\dot{x}(t) = A_2(x(t))\hat{x}(x(t)) + B_2(x(t))u(t),$$

where

$$A_1(x(t)) = \begin{bmatrix} 5 & 1.2x_1^2 + 0.1 \\ 2.5 & -3 \end{bmatrix}, \quad B_1(x(t)) = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

$$A_2(x(t)) = \begin{bmatrix} 5.4820 & 1.2x_1^2 + 0.1 \\ 2.5 & -3 \end{bmatrix}, \quad B_2(x(t)) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

The membership functions are defined as follows:

$$h_1(x(t)) = 1 - \frac{1 + e^{-4}}{1 - e^{-4}} \cdot \frac{1 - e^{-(x_1+2)}}{1 + e^{-(x_1+2)}},$$

$$h_2(x(t)) = \frac{1 + e^{-4}}{1 - e^{-4}} \cdot \frac{1 - e^{-(x_1+2)}}{1 + e^{-(x_1+2)}}.$$

Using the stabilization results developed in [41] for polynomial fuzzy systems, we have the following state-feedback controllers:

$$K_1(x(t)) = \begin{bmatrix} -4.4556x_1^2 - 0.1889 \times 10^{-8}x_1x_2 + 0.5375 \times 10^{-9}x_1 \\ -0.5657x_2^2 - 0.2960 \times 10^{-8}x_2 - 9.4618, \\ 0.2159x_1^2 + 0.3855 \times 10^{-9}x_1x_2 + 0.3565 \times 10^{-9}x_1 \\ -0.0966x_2^2 + 0.4307 \times 10^{-9}x_2 + 2.0554 \end{bmatrix},$$

$$K_2(x(t)) = \begin{bmatrix} -4.4556x_1^2 - 0.5937 \times 10^{-9}x_1x_2 + 0.1798 \times 10^{-9}x_1 \\ -0.5657x_2^2 + 0.7305 \times 10^{-9}x_2 - 9.7894, \\ 0.2159x_1^2 + 0.1642 \times 10^{-10}x_1x_2 + 0.2574 \times 10^{-9}x_1 \\ -0.0966x_2^2 - 0.2666 \times 10^{-10}x_2 + 2.0881 \end{bmatrix}.$$

Set the initial conditions of states as $x(t) = [-1 \ -2]^T$. Fig. 7 plots the state response of the closed-loop system with the polynomial state-feedback controller, and the control signal of the controller is shown in Fig. 8. It is shown that the closed-loop system goes to stable at about 2.8s.

Remark 12 In some situations, the initial conditions will influence the stability of the control systems. In this paper, the stability analysis is based on Lyapunov stability theory. As presented in the definition of Lyapunov stability, the system will get stable for any initial conditions. If the system is globally stable, there will be no such influence on system stability, i.e., the system is stable for any initial conditions. If it is not, the basin of attraction can be investigated but would be a more complicated problem.

5 Conclusions

In this paper, the stabilization problem for nonlinear systems has been investigated through the polynomial-approximation-based approach. By using the polynomial approximation functions, the nonlinearities existing in the nonlinear system have been approximated with approximation errors. And the errors are treated as modeling uncertainties. The nonlinear system is represented by

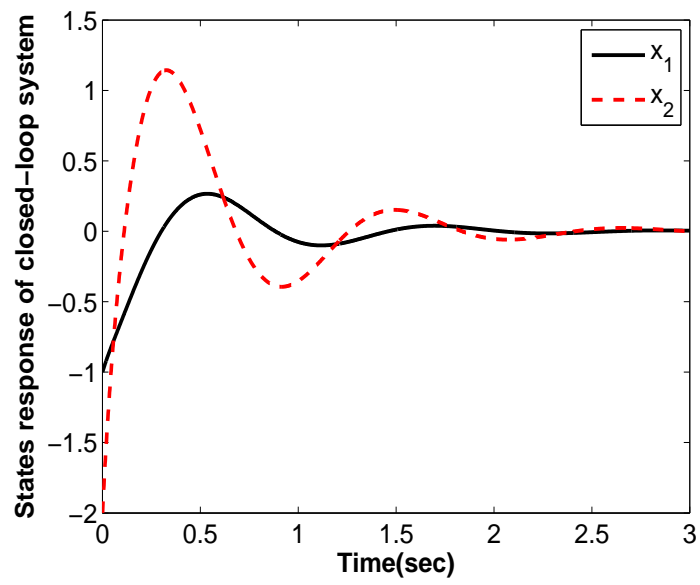


Fig. 7 The state response.

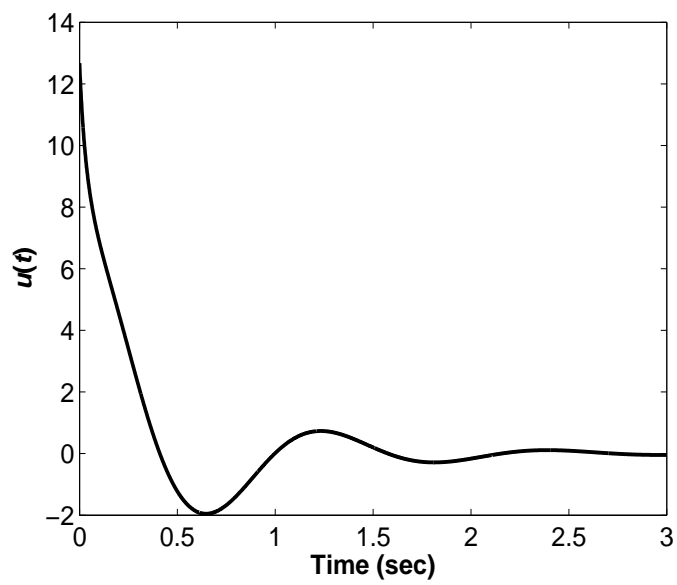


Fig. 8 The control signal.

the polynomial system with uncertainties. By using the piecewise polynomial approximation functions, the approximation accuracy is enhanced. Furthermore, the existence conditions of the controller design are obtained, which guarantee the resulting closed-loop systems is stable. Finally, simulation results are provided to illustrate the effectiveness of the method proposed in this paper and the advantages over those of the fuzzy-model-based approach. In our future work, we will consider the sliding mode control problem [19, 22, 28, 29] for nonlinear systems based on polynomial-approximation-based approach.

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